Counting Months and Years: The Upper Back Dial of the Antikythera Mechanism[1]

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Introduction

Toothed gearing has a natural application to instrument-making in the provision of a counting mechanism. I show here how a counting function is embedded within the design of the oldest surviving geared device, and certainly one of the earliest intricate scientific instruments known, the Antikythera Mechanism.

The general arrangement of this instrument, with a principal dial system on the 'front' face and two further dial systems, one above the other, on the 'back' face, is illustrated in an earlier paper in this journal.[2] In that paper I adumbrated a new gearing scheme for the instrument which I presented in a further paper.[3] There I offered, in a somewhat compressed form, corrections to the previously-accepted arrangement of the surviving wheels as well as revised figures for the numbers of teeth in each, based on my analysis of radiographs of the original fragments of the Antikythera Mechanism prepared by the late Allan Bromley and myself. In these respects the work of Derek de Solla Price, on which all previous reconstructions of the Antikythera Mechanism are based, is superseded.[4] I also suggested reconstructions of lost parts of the gear trains. In particular, the reader may have noticed that I offered a completion of the train leading to the two pointers of the upper back dial, which Price did not do. My present purpose is to expand on this feature.

The essential problem with the upper back dial is that, due to the way in which the instrument broke up, the gear train leading to it is incomplete. Part of the train can be traced within Fragment **A**, from wheel B2 – which rotates under the centre of the front dial with a period representing one year – through axis L to axis M at the edge of the fragment, as indicated in Figures 1 and 2.[5] The surviving piece of the upper back dial itself constitutes most of Fragment **B**, as seen in Figures 3 and 4. At axis N, its centre, there is the stub of an arbor but no wheel; at axis O, the centre of its subsidiary dial, there is an arbor bearing the wreck of a single wheel and what is probably a tiny fragment of the frame plate on which most of the wheelwork was planted.[6] I suggest that there must have been a further intermediate arbor, carrying two wheels to complete a compound train transmitting motion from a second wheel on axis N to the wheel on O. By analogy with the arrangement of the train behind the lower back dial, we would expect the arbor to have pivots working in holes both in the frame plate and in the dial plate, but the remains of these parts do not extend far enough to provide firm evidence.

A reconstruction of the train as far as the upper back dial centre, axis N, depends on estimating the numbers of teeth in the remaining wheels, but it also depends crucially on working out the correct relationship between Fragments **A** and **B** in order to judge what is likely to have been lost between them. The way in which the fragments come together leaves little room for doubt that a wheel on the arbor at N was engaged directly by pinion M2, with no intermediate axis. Price got this far, but then he fumbled: having positioned the centre of the upper back dial correctly in relation to Fragment **A**, he then suggested restoring to axis N the detached wheel that he found in Fragment **D**. This wheel is actually far too large to fit, but perhaps Price felt compelled to find a place for it somewhere in his reconstruction, in which he imagined adding very little to what survives. Seeing, however, that the gearing scheme must have been more extensive than he supposed (as I have argued previously: see note 2), we need have no inhibition in suggesting that Fragment **D** may well have come from some other part of the instrument. It may indeed not have been a part of this instrument at all.

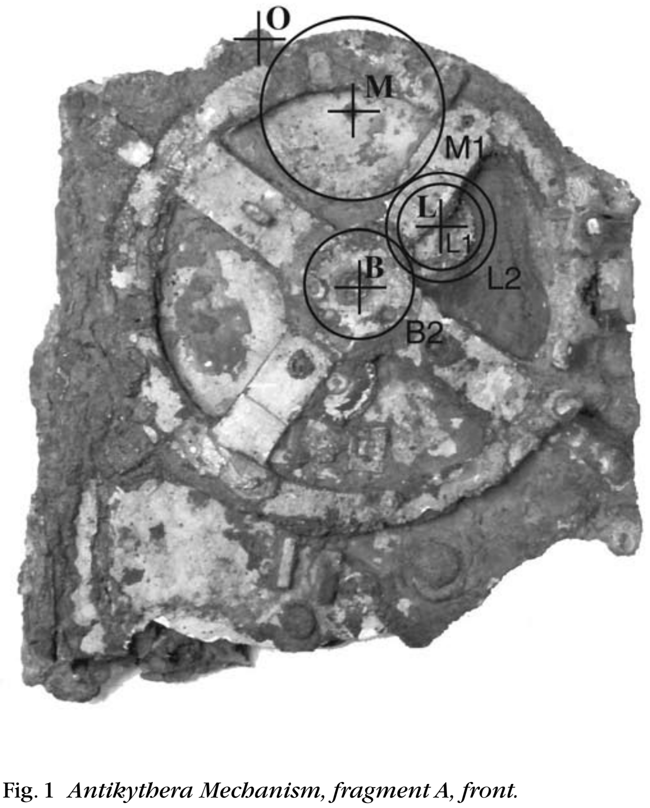

Fig. 1 *Antikythera Mechanism, fragment A, front. Approximately 50% actual size.*

Fig. 2 *Antikythera Mechanism, fragment A, back. Approximately 50% actual size.*

Fig. 3 *Antikythera Mechanism, fragment B, outside. Approximately 90% actual size.*

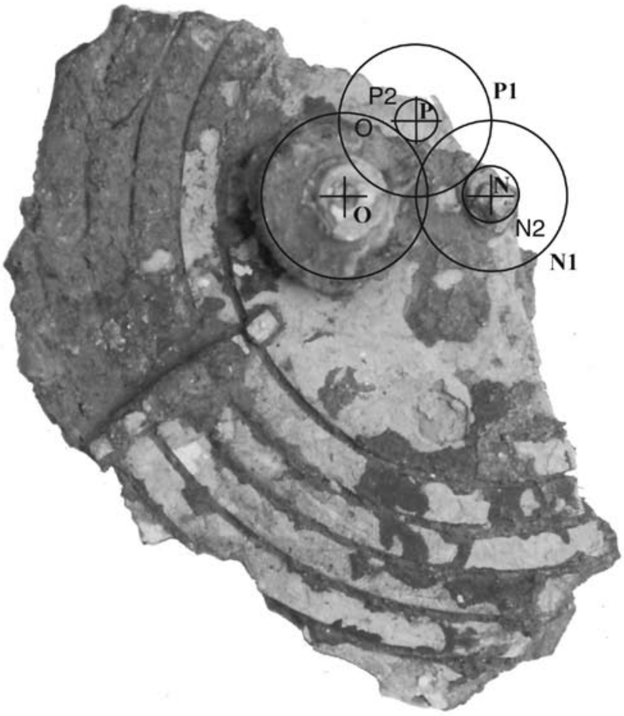

Fig. 4 *Antikythera Mechanism, fragment B, inside. Approximately 90% actual size.*

In consequence of his use of a wrong wheel at axis N, and the limitations of his estimation of the numbers of teeth in the wheels on axes L and M, Price's suggestions for the period represented by the rotation of the pointer at the upper back dial were ill-founded. He came tantalisingly close to what I will show to be the correct result, but with wrong numbers for the wheels. Finally, however, he seems to have decided that one turn of the pointer probably represented the passage of four years. In his attempt to interpret the function of the dial as a whole, Price was led further astray by his mistaken observation of two wheels at axis O.

There have been other attempts to make sense of this dial. Since, however, none has been based on any evidence other than that found in Price's published paper, and most depend on distorting that evidence in ways that it will not bear, I pass over them in silence and offer my own account.

Reconstructing the Gear Train to Axis N

The Table lists the numbers of teeth of those wheels that are relevant to the present discussion. It is abstracted from the larger table given in my last paper (note 3), to which the reader is referred for an explanation of how the figures were arrived at and for comparison with the figures previously offered by Price. As in that paper, figures that are certain are printed in bold typeface while the degree of uncertainty in the counts of other surviving wheels is expressed by the range in the right-hand column, and conjectural figures for wheels that are altogether lost are printed in italic typeface. Note, in addition, that 97 fits the observed data for wheel M1 slightly less well than 96, and 95 and 98 are less likely still.

In determining the period represented by one turn of the main pointer of the upper back dial at axis N, we are concerned only with the first six rows of figures, remembering that one turn of wheel B2 represents one year. The adoption of 53 teeth for the conjecturally-restored wheel N1 is, in the first instance, based on an attempt to judge its size, and for this we need to consider the correct juxtaposition of Fragments **A** and **B**.

Fragment **E**, which is roughly lozenge-shaped and measures about 60 by 35 by 14 mm., is seen under radiography to contain small portions of the back dial plate, including small parts of the two outermost turns of the spiral system of the lower back dial. Its significance for our present purpose is that, resting between Fragments **A** and **B**, it confirms the fit of one to the other, as shown in Figure 5. The circular lobe that is preserved almost miraculously in the corrosion products at the upper edge of Fragment **A** beyond the crumbled margin of the frame plate (Figures 1 and 2) is seen to be a relic of axis O, probably a washer similar to those found where other arbors take bearing in the plate. The break that formed the edge of Fragment **B**, running downward and slightly to the left through axis N (as seen from the outside) is seen to continue in Fragment **A** as the break that detached the left half of the epicyclic platform.

The relationship can also be explored further using radiographs. Axes G (centre of the lower back dial), B (centre of the front dial), and M in Fragment **A** are seen to lie on the vertical midline of the instrument. Axis N in Fragment **B** is found to lie on the same midline. Fragment **B** fits with the inner end of the spiral system around the upper back dial also lying on this midline, and with the line joining axes N and O at right-angles to the midline. These exercises yield a centre-distance between axes M and N of about 18.2 mm.

I proceed from this estimate of the separation of arbors M and N, and from the

Fig. 5 *Antikythera Mechanism, fragments A, B and E together. Approximately 59% actual size.*

measurement of what remains of pinion M2, to calculate the size and number of teeth of the wheel N1 that it drove; but I do so with some diffidence. The form of the surviving teeth throughout the mechanism is crude, with roughly straight flanks and pointed tips, appearing to be merely the result of cutting out the spaces between them using a file having an edge with an included angle of about 60º.[7] Moreover, while much of the observable irregularity in their form may result from damage, the markedly irregular spacing that is found in many places must be largely a feature of their original manufacture. Under these circumstances, discussions of pitch and the pitch circle, on which such a calculation is based, become rather vague.

Therefore a fairly crude calculation is all that is appropriate, and it runs as follows. The tip and root radii of pinion M2 are about 4.5 and 3.5 mm. respectively, so the radius of its pitch circle is roughly 4.0 mm. The corresponding radius of the pitch circle of N1 would therefore be about (18.2 – 4.0) = 14.2 mm., the pitch of wheel N1 should equal that of pinion M2, and so the number of teeth in wheel N1 should be the integer closest to (15 x 14.2/4.0), which suggests a wheel of 53 teeth. Even with well-divided gears having teeth of a more sophisticated form, the number of teeth would still not be very closely defined because the small pinion M2 leads. In such a case, and especially where the load on the train is light and uniformity of lead does not matter, a certain latitude in both pitch and size of the wheel may be permissible.

However, a count of 53 teeth for wheel N1, taken together with the certain and most probable tooth-counts tabulated for the other wheels, gives a velocity ratio for the train from axis B to axis N of 3.8: 1. That is, one revolution of the pointer on the upper back dial represents 3.8 years or, according to the 19-year period relation that is already known to be embodied elsewhere in the instrument, 47 synodic months.[8] This result fits with, and makes sense of, all the available evidence in providing a satisfactory solution to the question of the function of the upper back dial.

The Dial Plate

I have drawn attention to the spiral design of the back dial systems (note 2). The arrangement can now be better illustrated by reference to my reconstruction of the dial plate, which is shown in Figures 6 and 7. The graduations of the upper and lower back dials form spiral scales of five and four turns respectively, accompanied by spiral slots through the plate. The slots are designed to hold moveable markers, riveted loosely into them so that they may be moved along the scales at will. It is not altogether certain that the two spirals should actually meet in the middle to form a continuous S-curve as seen here, but analysis of the geometry and dimensions of the surviving fragments leads to this as a likely arrangement. The detail is probably unimportant, but it would have allowed the user to slide markers from one dial system to the other. The inside view, Figure 7, shows the form of the strips that are riveted across the slots to hold the spiral in shape, attested by two surviving examples in the original. These form little bridges that leave the edges of the slots clear for the passage of the rivet-heads.

A period of revolution for the main pointer on the upper back dial representing 47 synodic months is compatible with Price's observation, confirmed by Bromley and me[9], that the visible graduations on the fragmentary upper back dial suggest that the full circle was divided into 47 or 48 parts. I accept 47 divisions as the correct number. The dial system, with its 5-turn spiral, thereby provides a scale of 235 month divisions of a reasonable size, according with the maker's known interest in the 19-year cycle of 235 synodic months. The division of each turn of the spiral into an exact number of parts also agrees with both direct and radiographic observations that the dial divisions run radially straight across the several turns of the spiral.

It is evident that characters were engraved in many – perhaps all – of the spaces between the dial divisions. Presumably the lettering comprised numerals or abbreviations, but too little has been read for any attempt to be made to restore its sense.[10]

The Subsidiary Dial

It is useful to consider first what we know about the subsidiary dial of the lower back dial system. Here the subsidiary pointer (at I) was worked from the main pointer (at G) through a two-stage compound train planted at axes G, H and I. Parts of all the wheels on these axes survive, and although the tooth-counts of some are uncertain we find that the lower subsidiary pointer turned at roughly one-twelfth of the speed of the main one. In discussing that train I argue that a slow-turning subsidiary pointer makes sense only if its period is a simple multiple of the period of the main pointer, and so the only possible value of the ratio in that instance is exactly 12: 1.[11]

I agree with Price in reading the letter H, engraved within the exposed part of the bounding circle of the lower subsidiary dial, to the lower left, although whereas his sketch shows it set horizontally I see it with its upright strokes aligned to the radius of the circle at that point. Looking obliquely under the overlying layer, I see parts of other engraved letters that Price missed: to the right, slightly above the centre, I see an angular corner where two strokes meet, with a serif; to the upper left I see two strokes which appear to terminate in bold serifs, very like part of another H, aligned roughly to radii of the circle. I read these incomplete traces as letters Δ (to the right) and IB (to the upper left) respectively, and I interpret them as numerals: Δ = 4, H = 8 and IB = 12. No radial divisions are visible but it is notable that the dial plate is broken along a fairly straight line, vertically downward from the centre of the subsidiary circle, which may indicate fracture along a scribed radial division. Even without division lines, the numerals seem to indicate a tripartite division of the subsidiary dial, and to denote the cumulative number of draconitic months represented as having elapsed (equal to the number of turns made by the main pointer) as the subsidiary pointer sweeps out each third of its circle.

Now we turn to the subsidiary dial of the upper back dial system. Analysis of the fragmentary wheel on axis O, at the centre of the subsidiary dial, yields an uncertain count of 60 teeth. This wheel could have been driven directly by a second wheel on the arbor at N, but then the subsidiary pointer would have rotated just a little slower than the main one, one turn representing about $4^1/_2$ years, and in the opposite sense. This is not a useful function, and throughout the rest of the instrument the designer seems to show a preference for pointers rotating clockwise with advancing time. In any case the subsidiary pointer of the lower back dial certainly turned in the same sense as the main one.[12] Uniformity of the sense of rotation would most simply have been preserved by driving the arbor at O from the arbor of the main pointer at N through a compound train involving a further axis, in just the same way as the arbor at I was driven from the arbor at G through wheels on axis H. This arrangement offers us scope to reconstruct a useful function for the subsidiary dial. None of these points alone makes a strong argument, but I will show that, together with the evidence of the dial markings themselves, they do lead to a compelling solution.

Therefore I introduce the intermediate axis P as a conjectural reconstruction. The high number for wheel O dictates that it, and with it the pointer on the upper subsidiary dial, rotated more slowly than the arbor at P and so, almost certainly, more slowly than the corresponding main pointer on axis N. It remains to determine the appropriate velocity ratio for the upper back dial, between axes N and O.

I agree with Price that the subsidiary dial in the upper system was divided into four, as described below. In the lower system, in which the velocity ratio between the two pointers was 12: 1, the subsidiary pointer moved on a third of a turn, from one mark to the next, each time the main one swept out the whole of its four-turn spiral scale. Applying the same principle to the upper system, with the subsidiary dial divided into four and the main scale of five turns, the velocity ratio between the two pointers would have been 20: 1. One revolution of the subsidiary pointer would then represent the passage of 20 x 3.8 = 76 years.

This output can be achieved using pinions of 12 and 15 and two wheels of 60, as shown in the Table. Other numbers could of course be used, but this extra wheelwork fits neatly, adopting pitches within the extremes found elsewhere in the fragments and with axis P planted above and between axes N and O. Just possibly a notch in the upper edge of Fragment **B** may be interpreted as a broken-out pivot hole in the dial plate, but this is uncertain. The corresponding part of the frame plate is broken away.

Price shows the subsidiary dial divided into four quadrants, with the letter Σ in the lower left quadrant. He must, however, have muddled his notes, because this part of the circle is overlaid by accretions; only the upper left part of the circle is visible. Two radii are visible, and although the observation is difficult I take them to be drawn at right-angles, horizontally and vertically on the plate. Within this quadrant I do indeed see traces of a letter, which I initially read as Σ but which Bromley read as A or Δ. All that it seems safe to say is that we both saw straight strokes that did not meet at right-angles. In tomographic radiographs which resolve the plane of the dial plate, in this region I see, tentatively, the two letters ΛH, aligned as before upright to the radius. In the lower left quadrant I see more clearly the letter I, followed by another, indistinct letter. I suggest that the readings, from lower left quadrant and going clockwise, should be: IΘ, ΛH, NZ and OϚ. (The unfamiliar last character, 'stigma' or 'digamma', is used to denote the number 6. It appears in an inscription noted below in the form of a 'square **C**' with heavy serifs.) These are again interpreted as numerals: 19, 38, 57 and 76 respectively. By analogy with the lower subsidiary dial, these indicate the cumulative number of some period of interest represented as having elapsed as the pointer of the main dial runs over the whole of its spiral scale and as the subsidiary pointer traverses each sector of the circle in turn; but this time the period of interest is one year whereas the period represented by one turn of the main pointer is 3.8 years.

By direct inspection one may see some slight, rather uneven graduations outside the bounding circle of the subsidiary dial; and in radiographs I see indistinctly what may be lettering in this position. The spacing of the marks is such that there might be 20 around the circle, in which case they would serve as indicators for individual turns of the main pointer. My impression is that these are marks of a type that all who are familiar with instruments will recognize, those added by the user for his own convenience. I have made no attempt to reproduce them on my model.

Conclusion

I conclude by showing that this restoration of the display of the upper back dial possesses a coherence and usefulness that lend weight to its plausibility.

Just as five turns of the main pointer, sweeping out the whole length of the spiral scale, represents the passage of the Metonic period of 19 years, so one turn of the subsidiary pointer represents 76 years, an interval of time known to astronomers in antiquity as the Callippic Period. The significance of this period, four times the length of the other, is that, while it preserved the same excellent period relation between the synodic month and the year, it was reckoned to contain exactly 27759 days so that it embodied good approximations to the lengths of the synodic month and of the year in days. It seems also that astronomers found it a convenient period for the reckoning of long intervals of time: thus, in the *Almagest*, in giving the date of observations recorded by his predecessors, Ptolemy refers to day and month in the Egyptian calendar (in which, with its year of 365 days, it was understood that dates drifted by one day every four years in relation to the seasons) and a numbered year within a given Callippic cycle. On the instrument, while the main pointer of the upper back dial showed the correspondence of synodic months to years, and the rotation of the subsidiary pointer indicated the passage of Metonic and Callippic cycles, the end of one month, year or cycle and the beginning of the next would be indicated with more precision on the front dial. Thus, the upper back dial system could function as a convenient counter of months, years and cycles of years, in support of the display on the front.

Whether or not one accepts my reconstruction of it as a full planetarium, it is certain that the front dial display entailed some epicyclic modelling and was therefore relatively elaborate. One may envisage the user exploring its indications of astronomical events for times rather distantly removed from his own. In casting a personal horoscope, for instance, the astrologer required the places of the planets at the native's moment of birth, and although we have abundant evidence of the use of tables for computation,[13] a planetarium instrument could have been used to find the data mechanically. Since it takes about

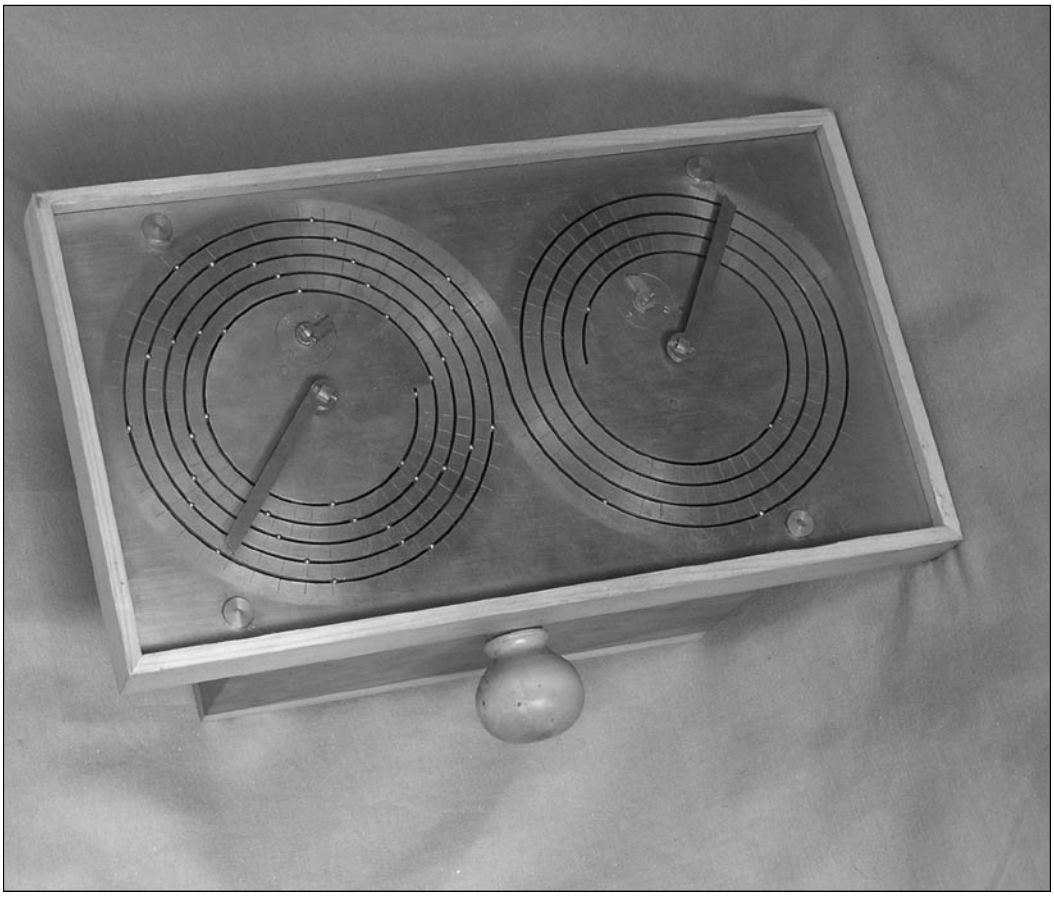

Fig. 6 *The Antikythera Mechanism, reconstruction by M.T. Wright: the back dial.*

five turns of the driving knob to move the display of this instrument by one year, it would have been a great convenience to have had the use of the upper back dial as a year-counter in working it through an interval corresponding to a person's age.

The upper back dial, used either alone or together with the front dial, might also be used in converting between any of the several luni-solar calendars used locally in the Hellenistic world for civil purposes and the Egyptian solar calendar, favoured by astronomers for its relative stability. The user might move the marker-beads along the slot to show any event of interest, such as the beginning of a new year or the astronomically-determined time of some festival. In any case it is interesting to note that the front dial, with its division into days and months according to the Egyptian calendar, and the upper back dial, with its count of months and years in the 19- and 76-year cycles, work together in just the same divided time-reckoning system that we find used by Ptolemy in the *Almagest*.

My conclusion that each turn of the main pointer of the lower back dial of the instrument probably represented one draconitic month, a function of use in attempting to predict eclipses (note 11), suggests a further possible use for the upper dial. Eclipse prediction is uncertain according to any simple astronomical model, but it was recognized that eclipse events kept roughly to a pattern that repeats after 223 synodic months, the so-called Saros cycle.[14] The spiral scale of 235 synodic months is long enough to contain one complete Saros cycle and a little more. The moveable marker beads might be set, according to the month graduations, to signal the eclipse possibilities of a whole cycle at a view.

Finally, there is further evidence from the inscription on what Price called the 'back door plate', a leaf of bronze that may or may not have been jointed to the case but which evidently lay over the back dial as the instrument decayed. Its extended inscription, apparently related to the function of the instrument, is now reduced to tantalising fragments. In one place characters in one line refer to both the 19-year and the 76-year cycles, while in the next line a possibly uncertain reading suggests 223 conjunctions.[15] The first line seems to refer to the periods displayed on the upper back dial; and in the reading of the second line we have a clear reference to the eclipse cycle.

Afterword

The arrangement and function of the upper back dial has in part been reconstructed by analogy with what is known about the lower back dial, and yet the interpretation of the upper dial now seems, if anything, more secure and more complete than that of the lower. It may be that the argument can now be reversed, and that our new insight into the design and function of the upper back dial may help us to gain a better understanding of how the indications of the lower back dial were meant to be read and used.

Taking my reconstruction of the back dial as a whole, and comparing it with that of Price, it will be seen that the indication of the synodic month, surely one of the more widely recognized and widely used astronomical periods, is much reduced in prominence. The alert reader of my last paper (note 3) will, however, have noticed that a new feature is added to the front dial. It appears at the top of my gearing diagram, but it was unfortunately slightly cropped in the final printing. To make good the omission, this part of the diagram is reproduced here as Figure 8. This element of my reconstruction, which is soundly based on evidence found in fragment C, supplies what seemed to have been lost in abandoning Price's: an easily-read indication of the synodic month. In addition, it provides a visual display of the Moon's phase. The feature forms the topic of a further short paper, now in preparation.

Acknowledgement

I am grateful to Dr Silke Ackermann of The British Museum and Dr J.V. Field of Birkbeck College, both of whom have allowed me to engage them in discussion which has been most helpful in clarifying the ideas that I have attempted to express here. Any mistakes are, however, entirely my own.

Table

Tooth-counts for the wheels in the train leading to the upper back dial.

B2	**64**	
L1	38	37 - 38
L2	**53**	
M1	96	95 - 98
M2	**15**	
N1	*53*	
N2	*15*	
O	60	57 – 62
P1	*60*	
P2	*12*	

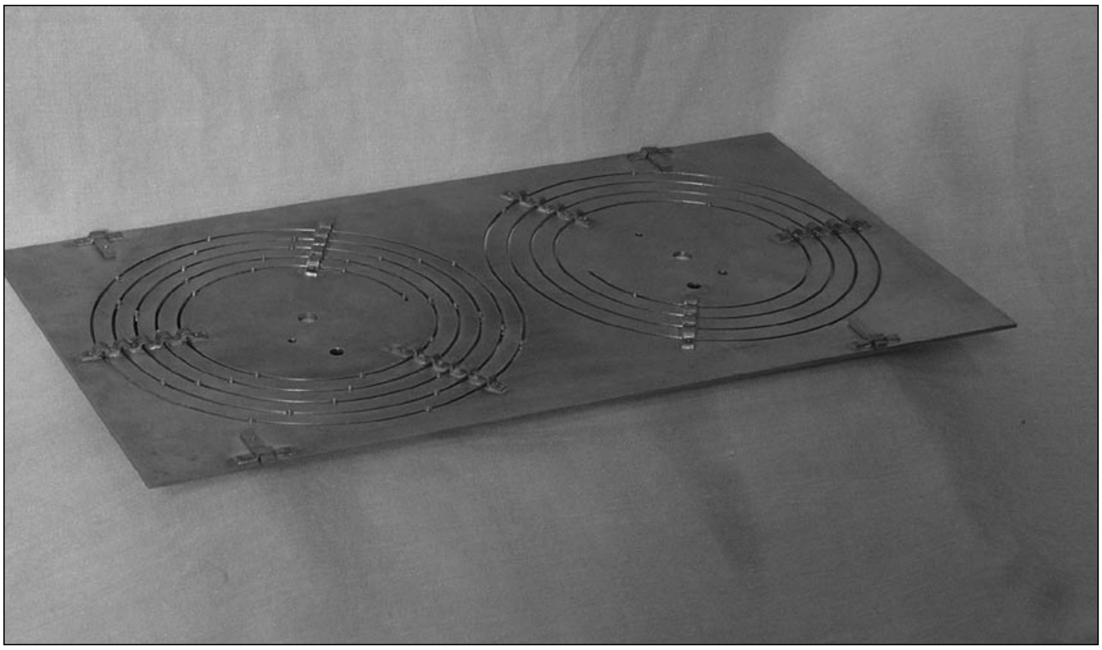

Fig. 7 The Antikythera Mechanism, reconstruction by M.T. Wright. Back dial plate, inside view.

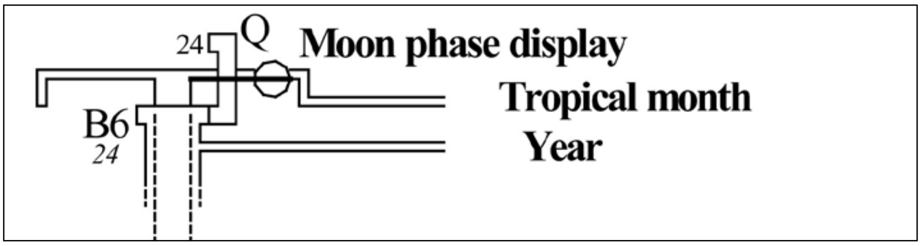

Fig. 8 *Top part of the reconstructed gearing diagram*

Notes and References

1. This paper was submitted in its original form in May 2005. It was to have appeared in issue no. 86 (September 2005) but was deferred to make way for material in memory of the late J.R. Millburn.

2. M.T. Wright, 'The Scholar, the Mechanic and the Antikythera Mechanism', *Bulletin of the Scientific Instrument Society*, no. 80 (March 2004), pp. 4 – 11.

3. M.T. Wright, 'The Antikythera Mechanism: a New Gearing Scheme', *Bulletin of the Scientific Instrument Society*, no. 85 (June 2005), pp. 2 – 7.

4. D.J. de S. Price, 'Gears from the Greeks', Transactions of the American Philosophical Society, **64** No.7, (1974); reprinted as an independent monograph, Science History Publications, New York 1975.

5. The designation of the fragments by alphabetical letter is already found in the earliest published description and was used by Price (note 4). The designation of wheels by alphabetical letter for the axis and Arabic numeral for the wheel is due to Price himself. Both are convenient, as is continuity within the literature. Therefore, despite a slight risk of confusion between the two systems, more apparent than real, I perpetuate the use of both systems, with some extension.

6. I find only one wheel on axis O. I cannot explain either how Price (note 4) can have persuaded himself that he saw two wheels (O1 and O2) or how he arrived at the tooth-counts that he offered for them.

7. Their form and appearance are easily reproduced by the use of a saw-file that has the section of an equilateral triangle. I suggest that this is probably very close to the method used by the maker of the instrument.

8. It will be recalled that the 19-year period relation, according to which 19 years = 235 synodic months (sometimes known in modern literature as the "Metonic" relation after the Athenian astronomer who used it as the basis for calendrical reform) is implied by the relation 19 years = 254 tropical months which is built in to the gearing behind the front dial.

9. Bromley's unpublished research notes are in my possession.

10. Bromley and I limited our attempts at reading inscriptions, but invited an experienced epigrapher to collaborate with us. His report remains in preparation.

11. M.T. Wright, 'Epicyclic Gearing and the Antikythera Mechanism', *Antiquarian Horology.* Part 1: **27** No. 3 (March 2003), pp. 270 – 279; Part 2: vol. 29 no. 1 (September 2005). pp.51 – 63.

12. Elsewhere (note 11, part 2) I point out the kinematic possibility that, if there were no idle wheel J within the epicyclic cluster on axis E, the uncertain numbers of teeth in the other wheels in the cluster might be chosen so that the pointers of the lower back dial turned anticlockwise; but I give several reasons for rejecting this possibility.

13. A. Jones, *Astronomical Papyri from Oxyrhynchus*, Philadelphia, American Philosophical Society (Memoirs vol. 233), 1999.

14. The modern application of the name *Saros* to this period is a mistake due to Edmund Halley.

15. See Price (note 4), Figs. 36 & 39 and p. 50. I bow to the experience of the epigraphers on whose work Price drew in describing the reading as 'uncertain and conjectural', but I read ΣΚΓ (= 223) quite clearly.

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